EBERHARD KARLS UNIVERSITÄT TÜBINGEN

MATH. - NATURWISS. FAKULTÄT
FACHBEREICH INFORMATIK
KOGNITIVE SYSTEME · PROF. A. ZELL

Mobile Robots

Summer Semester 2013
Assignment 8

due date: 25.06.2013, presentation: 02.07.2013
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Exercise 1 Kalman Filter (6 Points)

Assume the following basic linear system:

$$x_t = x_{t-1} + 2u_t + N(0, 0.5^2) \tag{1}$$
$$z_t = 0.5x_t + N(0, 0.3^2) \tag{2}$$

The first few control inputs and measurements are $u_1 = 2, u_2 = 1, z_1 = 4, z_2 = 7$. Starting with an initial estimate of $\hat{x}_0 = N(1, 0.5^2)$, compute two iterations of the Kalman Filter algorithm. Please provide all intermediate results for $\overline{\mu_i}$, $\overline{\Sigma_i}$, K_i, μ_i, and Σ_i.

Exercise 2 Linearization of a Measurement Model (7 Points)

For your flying robot, you want to use measurements of a barometric pressure sensor within an Extended Kalman Filter that estimates the robot's position. Use the following formula for barometric pressure:

$$p = p_0 \exp\left(\frac{-gMa}{RT_0}\right)$$

Here, a is the altitude in meters and $p_0 = 101\,325\,\mathrm{Pa}$, $g = 9.806\,65\,\mathrm{m/s^2}$, $M = 0.028\,964\,4\,\mathrm{kg\,mol^{-1}}$, $R = 8.314\,47\,\mathrm{J\,mol^{-1}\,K^{-1}}$, $T_0 = 288.15\,\mathrm{K}$ can be assumed to be constants for this exercise.

(a) Linearize the measurement function $z_t = h(x_t)$. Assume that $x_t = a_t$ and $z_t = p_t$. What is the general measurement Jacobian H for this example? Be careful with the units. (3 Points)

(b) Assume the mean of your latest pose estimate was $\mu_{t-1} = 50\mathrm{m}$. What is the measurement Jacobian H linearized around this point? What is H linearized around $\mu_{t-1} = 15k\mathrm{m}$? (2 Points)

(c) The datasheet of your pressure sensor says its noise can be assumed normally distributed with standard deviation $\sigma_p = 6\,\mathrm{Pa}$. Using the linearized measurement model from (b), how does this translate to uncertainty in altitude? Compute σ_a for both altitudes from (b). (2 Points)

Exercise 3 Linear and Nonlinear Systems (7 Points)

Assume a satellite orbits around earth (which is located in the origin) on a circular trajectory with a fixed and known angular velocity ω, but unknown radius r. In each timestep, the satellite can use its propulsion system to actively change its radius by a commanded $u_t = \Delta r$. (Note: This is a toy example. Disregard the fact that changing r would also lead to a different ω for a real satellite).

(a) Describe this as a linear discrete-time system. Choose $x_t = r_t$ as the system's state. What are A_t and B_t in this case? (2 Points)

(b) Again, describe this as a linear discrete-time system, but now use the satellite's cartesian coordinates as the system's state: $x_t = (x_t, y_t)^T$. What are A_t and B_t now? (3 Points)

(c) Assume the satellite can measure the radius r of its current trajectory, i.e. $z_t = r_t$. Are the systems from (a) and (b) still linear? Why? (1 Points)

(d) If the satellite can instead measure its position in cartesian coordinates, i.e. $z_t = (x_t, y_t)^T$, are the systems from (a) and (b) still linear? Why? (1 Points)