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MATH. - NATURWISS. FAKULTÄT
FACHBEREICH INFORMATIK
KOGNITIVE SYSTEME · PROF. A. ZELL

Mobile Robots

Summer Semester 2013
Assignment 3

due date: 14.05.2013, presentation: 28.05.2013
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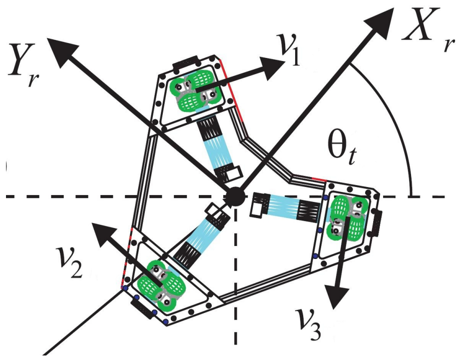
Exercise 1 (5 Punkte)



You are given an omni-directional robot with the following inverse kinematics:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & -l \\ 0 & 1 & -l \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & -l \end{pmatrix} \begin{pmatrix} {}^R v_x \\ {}^R v_y \\ {}^R \omega \end{pmatrix}$$

(a) Each wheel motor can reach a maximum rotational speed of $n = 444\,\mathrm{min}^{-1}$ rotations. The wheels have a diameter of $d_R = 80\,\mathrm{mm}$ and a distance from the center of $l = 185\,\mathrm{mm}$. Given these values, determine the maximum forward velocity of a single wheel v_{max}.

(b) Determine the maximum velocity of the robot towards the direction of its x-axis, its y-axis, and its maximum angular velocity.

Exercise 2 (6 Points)

You are given odometry values $U = (u_1, u_2, u_3, u_4, u_5, u_6)^T$. Estimate the trajectory of the following robots. The first state $x_0 = (0, 0, 0)^T$ is known for every robot.

(a) A differential drive robot with $l = 0.2\,\mathrm{m}$.
$U = ((0.55\pi, 0.45\pi)^T, (0.4\pi, 0.6\pi)^T, (2, 2)^T, (0.45\pi, 0.55\pi)^T, (1, 1)^T, (0.9\pi, 1.1\pi)^T)$.
$u_t = (s_{lt}[m], s_{rt}[m])^T$.

(b) A car-like robot with $l_w = 0.3\,\mathrm{m}$ and $l_a = 0.5\,\mathrm{m}$.
$U = ((-0.5, 0.5\pi)^T, (1, 0.5\pi)^T, (0, 2)^T, (0.5, 0.5\pi)^T, (0, 1)^T, (0.5, \pi)^T)$.
$u_t = (\psi_t, \Delta s_t[m])^T$.

(c) An omni-directional robot with $l = 0.2\,\mathrm{m}$.

$$U = \left(\begin{pmatrix} 0.25\sqrt{3}\pi + 0.1\pi \\ 0.1\pi \\ -0.25\sqrt{3}\pi + 0.1\pi \end{pmatrix}, \begin{pmatrix} 0.25\sqrt{3}\pi - 0.2\pi \\ -0.2\pi \\ -0.25\sqrt{3}\pi - 0.1\pi \end{pmatrix}, \begin{pmatrix} 1\sqrt{3} \\ 0 \\ -1\sqrt{3} \end{pmatrix}, \begin{pmatrix} 0.25\sqrt{3}\pi - 0.1\pi \\ -0.1\pi \\ -0.25\sqrt{3}\pi - 0.1\pi \end{pmatrix}, \begin{pmatrix} 0.5\sqrt{3} \\ 0 \\ -0.5\sqrt{3} \end{pmatrix}, \begin{pmatrix} 0.5\sqrt{3}\pi - 0.2\pi \\ -0.2\pi \\ -0.5\sqrt{3}\pi - 0.2\pi \end{pmatrix} \right) \quad (1)$$

$u_t = (v_1[\frac{m}{s}], v_2[\frac{m}{s}], v_3[\frac{m}{s}])^T$.

Exercise 3 (6 Points)

You are given the following trajectory:

$$t = 0 : x_0 = (0\,\mathrm{m}, 0\,\mathrm{m}, \frac{\pi}{2})^T \quad (2)$$
$$t = 1 : x_1 = (0\,\mathrm{m}, 1\,\mathrm{m}, \frac{\pi}{2})^T \quad (3)$$
$$t = 2 : x_2 = (-1\,\mathrm{m}, 2\,\mathrm{m}, \pi)^T \quad (4)$$
$$t = 3 : x_3 = (-2\,\mathrm{m}, 2\,\mathrm{m}, \pi)^T \quad (5)$$
$$t = 4 : x_4 = (-2\,\mathrm{m}, 3\,\mathrm{m}, 0)^T \quad (6)$$

Each of the following robots has to follow this trajectory. Estimate the necessary control commands u_t.

(a) A differential drive robot with $l = 0.2\,\mathrm{m}$. ($u_t = (v_{lt}, v_{rt})^T$)

(b) A car-like robot with $l_w = 0.3\,\mathrm{m}$ and $l_a = 0.5\,\mathrm{m}$. ($u_t = (v_t, \psi_t)^T$)

(c) An omni-directional robot with $l = 0.2\,\mathrm{m}$. ($u_t = (v_{1t}, v_{2t}, , v_{3t})^T$)

Exercise 4 (3 Points)

(a) Explain the axle-differential of car-like robots in your own words.

(b) A car-like robot is moving on an arc with radius $r = 1\,\mathrm{m}$. The model of the robot is specified by its parameters $l_w = 0.3\,\mathrm{m}$ and $l_a = 0.5\,\mathrm{m}$. Determine the axle-differential $d = s_l - s_r$ as function of Δt.