Mobile Robots

Summer Semester 2013
Assignment 1

Advisors: Sebastian Buck, Daniel Dube, Volker Grabe, Markus Ryll and Sebastian Scherer
due to: 30.04.2013, presentation: 07.05.2013

Exercise 1 (3 Points)

(a) Determine the homogeneous transformation matrix $\mathbf{R}_z(\alpha)$ for a rotation around the z-axis with angle α in 3D and the homogeneous transformation matrix $\mathbf{Trans}(\Delta x, \Delta y, \Delta z)$ about the vector $(\Delta x, \Delta y, \Delta z)^T$ in 3D.

(b) Now determine one homogeneous transformation matrix ${}^2\mathbf{T}_1$ which represents a rotation around the z-axis with the angle α and then a translation about the vector $(\Delta x, \Delta y, \Delta z)^T$.

(c) Transform a point ${}^1\mathbf{p} = (x, y, z)^T$ using ${}^2\mathbf{T}_1$.

Exercise 2 (5 Points)

(a) You learnt three different representations of probability distributions in the lecture. Name one advantage and one disadvantage for every of these representations. (3 Points)

(b) Name another continuous probability distribution besides the normal distribution and the uniform distribution. Provide the formula of the distribution, sketch its density function and briefly state its differences to the normal distribution. (2 Points)

Exercise 3 (5 Points)

The following set of samples $\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_6)$ with $\mathbf{s}_i \in \mathbb{R}^3$ is given:

$$\mathbf{S} = \left(\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \right) \tag{1}$$

(a) Estimate the mean μ_S of the distribution of $\mathbf{S}$.

(b) Estimate the covariance matrix Σ_S of the distribution of $\mathbf{S}$.

(c) Compute the value of $N(\mu_S, \Sigma_S)$ for the event $s = (1, 2, 3)^T$.

Exercise 4 (7 Points)

You are given the multivariate normal distributed random variable $x \in \mathbb{R}^2 \sim N(\mu_x, \Sigma_x)$, with mean $\mu_x = (1, 1)^T$ and covariance matrix:

$$\Sigma_x = \begin{pmatrix} 2 & 0.5 \\ 0.5 & 1 \end{pmatrix} \qquad (2)$$

This variable is now transformed by the following equations. Determine the parameters μ_y and Σ_y of the resulting normal distributed variable $y \sim N(\mu_y, \Sigma_y)$.

(a) $y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (1 Point)

(b) $y = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} x$ (1 Point)

(c) $y = \frac{x}{\Delta t}$ (2 Points)

(d) $y = x + \Delta t \cdot z$ with $z \in \mathbb{R}^2, z \sim N(\mu_z, \Sigma_z)$ (3 Points)